

### REMARKS

The Examiner has requested information to explain how certain formula listed in the patent were derived. Specifically, the Examiner requests that the Applicants identify any known publications that describe the business value creation, evaluation and calculations disclosed in the pending application on pages 12-16 and 29-30 of the pending application.

#### *Equations on pages 12-16*

Applicants derived the equations contained on pages 12-16 and 29-30 beginning with a conventional discounted present value calculation. Particularly, the conventional formula for computing the present value of \$1 due in  $n$  periods when the interest rate per period is  $i$  is given by the following equation:

$$V^n = \frac{1}{(1+i)^n}$$

The above formula is a conventional formula given in financial texts, such as the following:

***Financial Compound Interest and Annuity Tables, Fifth Edition***, Gushee, Charles, Financial Publishing Company, Boston, 1971. (See page 3, which is attached hereto as Exhibit A).

In the formula appearing on page 12 of the pending application, Applicants replaced the standard present value symbol at the left-hand side of the equation:

$$V^n$$

with the symbol ("InPV") to represent the projected after-tax cash inflows (Applicants later use the symbol "OutPV" on page 15 to represent the projected after-tax cash *outflows* — but the mathematical approach is identical). In the formula appearing on p.13 of patent application, Applicants also replaced the interest rate ( $i$ ) in the above formula with symbol  $ra\_atr$  referring to the risk-adjusted after-tax interest rate (as explained in the application). The "conventional present value formula" given above is for computing the present value of \$1  $n$  periods hence —

of course, it follows that if what is due  $n$  periods hence is \$2, then the present value is twice as much — in general, one multiplies the present value factor by the amount of the future cash flow. If that cash flow is  $CashIN_i$  (representing the projected after-tax cash *inflow* in year  $i$ ) then one multiplies the factor, as we have, by  $CashIN_i$ , as shown in the present value equation on page 12. These calculations may be repeated for each future year. Thus, for cash flows at the end of year 1 the standard formula

$$v^n = \frac{1}{(1+i)^n}$$

becomes

$$v = \frac{1}{(1+i)}$$

while for cash flows at the end of year 2, the standard formula

becomes

$$v^2 = \frac{1}{(1+i)^2}$$

Applicants expressed this repeated calculation for each successive year by using a summation formula

$$InPV = \sum_1^n CashIN_i \times \left( \frac{1}{1+ra_{atr}} \right)^i$$

indicating that the product for year  $i$

$$CashIN_i \times \left( \frac{1}{1+ra_{atr}} \right)^i$$

is to be summed over years 1 to n coupled with a probability factor to get an “expectation value”, i.e., outcome amount X probability of that outcome occurring. In the summation referred to above, the probability factor is added, so that the complete equation becomes:

$$InPV = \sum_i^n CashIN_i \times \left( \frac{1}{1 + ra\_atr} \right)^i \times InProb$$

where InProb is, as stated in the pending application, is the probability (as assessed by the user or by management of the business enterprise) of the *inflows* occurring.

Having provided formulae for inflows (InPV) and outflows (OutPV), using arithmetic, the net present value (NetPV) is equal to InPV - OutPV (the present value of inflows less the present value of outflows), as shown on page 14.

Finally, the last equation on page 14, TotNetPV = NetPV + RealOptVal, provides a TotNetPV value that is equal to the net present value (of inflows less outflows) increased by the “real options value (if any) included in any of the enterprise’s strategies.” The formula for computing the RealOptVal term, as stated in the patent application, “may be determined conventionally by reference to the Black-Scholes equation.” The Black-Scholes equation (which Black and Scholes published in 1973) is well-known to those skilled in the art, and is provided in standard financial texts, such as ***Black-Scholes and Beyond: Option Pricing Models***, Chriss, Neil A, McGraw Hill, New York, 1997 (see p. 152, which is attached hereto as Exhibit B).

#### *Equations on pages 29-30*

The first equation on page 29 is a variation of the present value formulae set forth above. The second equation on page 30 is a definition of “outcome variance” in a unique context, as explained in the pending application. Particularly, the variance compares a Scenario A case (A) to a base case (bc), each running from starting time  $t_1$  to ending time  $t_2$ .

The outcome variance ( $OutcomeVar_{A>bc}$ ) is defined as the excess of:

the total net present value for projected Scenario A at time  $t_2$

$TotNetPV_A$

over the present value of the base case at time  $t_1$

$TotNetPV_{bc}$

grown at a risk-adjusted after-tax rate of interest ( $ra\_atr$ ) from time  $t_1$  to time  $t_2$  — i.e.,  
multiplied by the factor:

$$(1 + ra\_atr)^{(t_2 - t_1)}$$

to yield:

$$OutcomeVar_{A>bc} = TotNetPV_A - TotNetPV_{bc} \times (1 + ra\_atr)^{(t_2 - t_1)}$$

The final equation on page 30 is the following:

$$OutcomeComp_{B>A} = TotNetPV_B - TotNetPV_A$$

This involves the definition of another type of outcome comparison — here called:

$OutcomeComp_{B>A}$

Instead of measuring an “outcome variance” comparing one Scenario at time  $t_2$  compared to a base case grown (from time  $t_1$  to time  $t_2$ ) at an assumed desired risk-adjusted after-tax interest rate (as in the first equation on page 30), one compares two “total net present values” at the same time.

To the extent that Scenario A is projected to be more valuable than Scenario B, the former’s TotNetPV will exceed the latter’s TotNetPV — and thus the outcome comparison is defined to be:

$$TotNetPV_B - TotNetPV_A$$

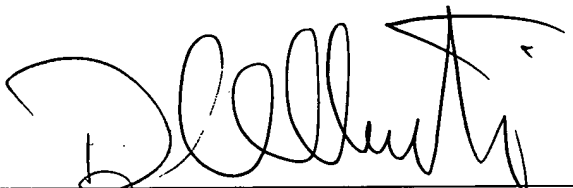
Early action on the present application is respectfully requested. In the event that matters remain to be resolved or if the Examiner has any questions, the Examiner is invited to contact Applicants' attorney at the following address or telephone number:

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Respectfully submitted,

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*and* ANNUITY TABLES

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*Great care has been taken to make these tables correct  
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PRESENT WORTH OF 1 <i>What \$1 due in the future is worth today.</i>	PRESENT WORTH OF 1 PER PERIOD <i>What \$1 payable periodically is worth today.</i>	PARTIAL PAYMENT <i>Annuity worth \$1 today. Periodic payment necessary to pay off a loan of \$1.</i>	P E R I O D S
.999 167 3605	.999 167 3605	1.000 833 3333	1
.998 335 4144	1.997 502 7749	.500 625 0868	2
.997 504 1609	2.995 006 9358	.333 889 0431	3
.996 673 5996	3.991 680 5353	.250 521 0503	4
.995 843 7298	4.987 524 2651	.200 500 2777	5
.995 014 5510	5.982 538 8161	.167 153 1152	6
.994 186 0626	6.976 724 8787	.143 333 7300	7
.993 358 2640	7.970 083 1427	.125 469 2055	8
.992 531 1548	8.962 614 2975	.111 574 5883	9
.991 704 7341	9.954 319 0316	.100 458 9060	10
.990 879 0016	10.945 198 0333	.091 364 2674	11
.990 053 9567	11.935 251 9899	.083 785 4116	12
.989 229 5987	12.924 481 5886	.077 372 5424	13
.988 405 9271	13.912 887 5157	.071 875 8057	14
.987 582 9413	14.900 470 4570	.067 111 9749	15
.986 760 6408	15.887 231 0977	.062 943 6303	16
.985 939 0249	16.873 170 1226	.059 265 6859	17
.985 118 0932	17.858 288 2158	.055 996 4084	18
.984 297 8449	18.842 586 0607	.053 071 2715	19
.983 478 2797	19.826 064 3404	.050 438 6540	20
.982 659 3969	20.808 723 7373	.048 056 7676	21
.981 841 1959	21.790 564 9332	.045 891 4215	22
.981 023 6762	22.771 588 6094	.043 914 3714	23
.980 206 8371	23.751 795 4465	.042 102 0803	24
.979 390 6782	24.731 186 1247	.040 434 7772	25
.978 575 1989	25.709 761 3236	.038 895 7325	26
.977 760 3986	26.687 521 7222	.037 470 6955	27
.976 946 2767	27.664 467 9989	.036 147 4509	28
.976 132 8326	28.640 600 8315	.034 915 4686	29
.975 320 0659	29.615 920 8974	.033 765 6223	30
.974 507 9759	30.590 428 8734	.032 689 9634	31
.973 696 5621	31.564 125 4355	.031 681 5368	32
.972 885 8240	32.537 011 2595	.030 734 2304	33
.972 075 7608	33.509 087 0203	.029 842 6513	34
.971 266 3722	34.480 353 3924	.029 002 0229	35
.970 457 6575	35.450 811 0499	.028 208 0993	36
.969 649 6161	36.420 460 6660	.027 457 0937	37
.968 842 2476	37.389 302 9136	.026 745 6177	38
.968 035 5513	38.357 338 4649	.026 070 6306	39
.967 229 5267	39.324 567 9915	.025 429 3957	40
.966 424 1732	40.290 992 1647	.024 819 4434	41
.965 619 4903	41.256 611 6550	.024 238 5392	42
.964 815 4774	42.221 427 1324	.023 684 6565	43
.964 012 1339	43.185 439 2664	.023 155 9530	44
.963 209 4594	44.148 648 7258	.022 650 7499	45
.962 407 4532	45.111 056 1789	.022 167 5147	46
.961 606 1148	46.072 662 2937	.021 704 8451	47
.960 805 4436	47.033 467 7372	.021 261 4559	48
.960 005 4390	47.993 473 1763	.020 836 1665	49
.959 206 1006	48.952 679 2769	.020 427 8911	50
.958 407 4277	49.911 086 7046	.020 035 6287	51
.957 609 4199	50.868 696 1245	.019 658 4555	52
.956 812 0765	51.825 508 2010	.019 295 5175	53
.956 015 3970	52.781 523 5980	.018 946 0238	54
.955 219 3809	53.736 742 9789	.018 609 2410	55
.954 424 0275	54.691 167 0064	.018 284 4882	56
.953 629 3364	55.644 796 3427	.017 971 1324	57
.952 835 3070	56.597 631 6497	.017 668 5838	58
.952 041 9387	57.549 673 5884	.017 376 2932	59
.951 249 2310	58.500 922 8194	.017 093 7474	60
$v^n = \frac{1}{(1+i)^n}$	$a_{\overline{n} } = \frac{1-v^n}{i}$	$\frac{1}{a_{\overline{n} }} = \frac{i}{1-v^n}$	$n$

RATE  
 $1/12\%$

.00083333  
per period

ANNUALLY  
If compounded  
annually  
nominal annual rate is

$1/12\%$

SEMIANNUALLY  
If compounded  
semiannually  
nominal annual rate is

$1/6\%$

QUARTERLY  
If compounded  
quarterly  
nominal annual rate is

$1/3\%$

MONTHLY  
If compounded  
monthly  
nominal annual rate is

$1\%$

$i = .00083333$   
 $j_{(12)} = .00166666$   
 $j_{(6)} = .00333333$   
 $j_{(12)} = .01$



# BLACK-SCHOLES AND BEYOND OPTION PRICING MODELS

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by the delta of the option. The conclusion to all of this is:

*The theoretical value of a vanilla European call option is completely determined by its delta, the risk-free rate of interest and the time to expiration.*

### Summary

Let's review the main points covered so far. First of all, we have shown that the Black-Scholes formula gives rise to a hedging strategy for the short position of a vanilla European call. To show that the value of the hedging portfolio is equal to that of the option at every time, we need to: 1) know the delta of the option at every time, and 2) use the delta to determine the correct value of  $B_t$  at every time  $t$ . There is only one thing left to do: give formulas for  $\Delta_t$  and  $B_t$ .

## 4.8 THE BLACK-SCHOLES FORMULAS FOR $\Delta_t$ AND $B_t$

Now that we understand the general idea of the Black-Scholes formula, let's actually see what it is. All we have to do is to give the formulas for  $\Delta_t$  and  $B_t$ .

The formulas are given in terms of the cumulative normal distribution function, discussed in Chapter 2. They are:

$$\Delta_t = N(d_1), \quad d_1 = \frac{\log(S_t/K) + (r + \frac{\sigma^2}{2})(T - t)}{\sigma \sqrt{T - t}} \quad (4.8.1)$$

$$B_t = N(d_2)K, \quad d_2 = \frac{\log(S_t/K) + (r - \frac{\sigma^2}{2})(T - t)}{\sigma \sqrt{T - t}} \quad (4.8.2)$$

where

$S_t$  = price of stock per share at time  $t$

$K$  = strike price

$r$  = risk-free rate of interest

$\sigma$  = volatility of stock under geometric Brownian motion model

$T - t$  = time until expiration

$N(\cdot)$  = cumulative normal distribution function

Combining this with equation (4.6.1), we present the Black-Scholes formula for vanilla European call options on a non-dividend-paying stock:

$$C_t = N(d_1) \cdot S_t - e^{-r(T-t)} K \cdot N(d_2). \quad (4.8.3)$$